

Review Questions for Exam 22
Linear Algebra

1. What matrix has the effect of rotating every vector through 60° and then projecting the result onto the y -axis?
2. What matrix represents projection onto the y -axis followed by reflection over the x -axis?
3. What matrix produces a stretching in the y -direction by a factor of 2? Then, Draw the circle $x^2 + y^2 = 1$ and its result from multiplication by the matrix from the previous part.
4. State the matrix for the given transformation and its result.
 - a. $(1, 2)$ after rotation by 30°
 - b. $(3, -2)$ after reflection over $y = 2x$
5. What 3 by 3 matrices represent the transformations that
 - a. project every vector onto the x - y plane?
 - b. reflect every vector through the x - y plane?
 - c. rotate the x - y plane through 90° , leaving the z -axis alone?
6. Suppose a linear T transforms $(1, 1)$ to $(2, 2)$ and $(2, 0)$ to $(0, 0)$. Find $T(v)$ when (a) $v = (2, 2)$. (b) $v = (3, 1)$. (c) $v = (-1, 1)$. (d) $v = (a, b)$.
7. (a) What matrix transforms $(1, 0)$ into $(1, 5)$ and transforms $(0, 1)$ to $(2, 2)$?
(b) What matrix transforms $(1, 5)$ to $(1, 0)$ and $(2, 2)$ to $(0, 1)$?
(c) Why does no matrix transform $(1, 5)$ to $(1, 0)$ and $(2, 10)$ to $(0, 1)$?

8. What matrix transforms $(2, 4)$ to $(1, 1)$ and $(1, 5)$ to $(0, 2)$?
9. Find the lengths and the inner product of $x = (1, 3, 0, -1)$ and $y = (0, -2, 7, -6)$.
10. Give an example in \mathbb{R}^2 of linearly independent vectors that are not orthogonal. Also, give an example of orthogonal vectors that are not independent.
11. Two lines in the plane are perpendicular when the product of their slopes is -1 . Apply this to the vectors $x = (x_1, x_2)$ and $y = (y_1, y_2)$, whose slopes are x_2/x_1 and y_2/y_1 , to derive again the orthogonality condition $x^T y = 0$.